

# Periodic Table of Mathematical Structures (PTMS)

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## Abstract

The Periodic Table of Mathematical Structures (PTMS) aims to systematically organize and classify mathematical structures by their levels of complexity and abstractness. Inspired by the periodic table of elements, this project seeks to create an ever-expanding framework that serves as an educational and research tool, highlighting the relationships and hierarchies among mathematical concepts.

## 1 Periodic Table of Mathematical Structures

| Abstractness<br>Complex-<br>ity | 1                                             | 2                                      | 3                                                     | 4                                                           | 5                                             | 6                                                                            |
|---------------------------------|-----------------------------------------------|----------------------------------------|-------------------------------------------------------|-------------------------------------------------------------|-----------------------------------------------|------------------------------------------------------------------------------|
| 1                               | Set: Distinct<br>objects                      | Relation:<br>Ordered pairs             | Function:<br>Mappings                                 | Tuple: Ordered<br>list of elements                          | Multiset:<br>Elements with<br>multiplicities  | Sequence:<br>Ordered list<br>with repetitions                                |
| 2                               | Ring: Two<br>operations                       | Module:<br>Generalized<br>vector space | Vector Space:<br>Vectors with<br>ops                  | Algebra: Vector<br>space with<br>product                    | Lie Algebra:<br>Non-associative<br>product    | Associative<br>Algebra:<br>Algebra where<br>multiplication is<br>associative |
| 3                               | Topological<br>Space:<br>Neighborhoods        | Manifold:<br>Locally<br>Euclidean      | Symplectic<br>Manifold:<br>2-form                     | Tensor<br>Category:<br>Tensor product                       | Fiber Bundle:<br>Space with a<br>projection   | Vector Bundle:<br>Vector space<br>parameterized<br>by another<br>space       |
| 4                               | Chain Complex:<br>Abelian groups              | Derived<br>Category:<br>Complexes      | Motivic<br>Homotopy:<br>Homotopy for<br>varieties     | Higher<br>Category:<br>Multiple<br>morphisms                | Exact Sequence:<br>Chain of maps              | Spectral<br>Sequence:<br>Sequence of<br>exact sequences                      |
| 5                               | Riemann<br>Surface: 1D<br>complex<br>manifold | Modular Form:<br>Analytic<br>function  | Automorphic<br>Form: Invariant<br>functions           | Noncommutative<br>Geometry:<br>Noncommuta-<br>tive algebras | Complex<br>Manifold:<br>Multi-<br>dimensional | Differential<br>Form:<br>Generalized<br>function for<br>integration          |
| 6                               | Graph: Vertices<br>and edges                  | Hypergraph:<br>Generalized<br>graph    | Matroid:<br>Generalized<br>independence               | Simplicial<br>Complex:<br>Generalized<br>graph              | Polytope:<br>Multi-<br>dimensional<br>shape   | Convex Hull:<br>Smallest convex<br>set containing a<br>set                   |
| 7                               | Category:<br>Objects and<br>morphisms         | Functor: Maps<br>between<br>categories | Natural<br>Transformation:<br>Functors of<br>functors | Monoidal<br>Category:<br>Tensor category                    | Closed<br>Category:<br>Functors as<br>objects | Higher<br>Category:<br>Multiple<br>morphisms                                 |

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| 8  | Partial Order:<br>Ordered pairs                               | Lattice:<br>Ordered structure                                          | Boolean Algebra:<br>Boolean operations                             | Heyting Algebra:<br>Generalizes Boolean algebra                  | Complete Lattice: All sup and inf                                       | Continuous Lattice: Lattice with certain completeness properties   |
| 9  | Propositional Logic:<br>True/False                            | First-Order Logic:<br>Quantifiers                                      | Modal Logic:<br>Necessity/Possibility                              | Intuitionistic Logic:<br>Constructivist approach                 | Higher Order Logic:<br>Quantifies over functions                        | Type Theory:<br>Computation with types                             |
| 10 | CW Complex:<br>Space decomposed into cells                    | Simplicial Complex:<br>Collection of simplices                         | Differentiable Stack:<br>Stack with smooth structure               | Orbifold:<br>Generalized manifold with singularities             | Stratified Space:<br>Space with stratification                          | Tame Space:<br>Space with tame topology                            |
| 11 | Azumaya Algebra:<br>Generalization of central simple algebras | Clifford Algebra:<br>Algebra associated with quadratic forms           | Witt Group:<br>Group of quadratic forms over a field               | Brauer Group:<br>Group classifying division algebras             | Quantum Torus:<br>Noncommutative deformation of the torus               | Quasi-Hopf Algebra:<br>Generalization of Hopf algebras             |
| 12 | Algebraic Surface:<br>Two-dimensional algebraic variety       | Calabi-Yau Manifold:<br>Ricci-flat Kähler manifold                     | K3 Surface:<br>Special type of Calabi-Yau manifold                 | Fano Variety:<br>Variety with ample anticanonical bundle         | Enriques Surface:<br>Algebraic surface with certain properties          | Abelian Variety:<br>Projective variety that is also a group        |
| 13 | A-Category:<br>Category with A-morphisms                      | E-Category:<br>Category with E-morphisms                               | Derived -Category:<br>Generalization of derived categories         | DG-Category:<br>Differential graded category                     | Infinity-Category:<br>Generalization of categories with infinite levels | -Category:<br>Higher category with morphisms between morphisms     |
| 14 | Hilbert Modular Form:<br>Generalizes modular forms            | Siegel Modular Form:<br>Modular forms for symplectic groups            | Automorphic Representation:<br>Representation of automorphic forms | Theta Function:<br>Special function of several complex variables | Moduli Space of Curves:<br>Parameter space for algebraic curves         | Hecke Algebra:<br>Algebra associated with double cosets of a group |
| 15 | Matroid Theory:<br>Generalizes linear independence            | Greedoids:<br>Generalizes matroids and antimatroids                    | Delta-Matroid:<br>Generalizes matroids                             | Chow Ring:<br>Intersection theory on algebraic varieties         | Graph Homomorphism:<br>Map preserving graph structure                   | Graph Minor:<br>Minor in a graph                                   |
| 16 | Infinity-Topos:<br>Generalization of topoi to -categories     | (,1)-Category:<br>Category with weak equivalences and higher morphisms | (,n)-Category:<br>Generalization of (,1)-categories to n-levels    | Double Category:<br>Category with two types of morphisms         | Tricategory:<br>Generalization of bicategories to three levels          | Pseudocategory:<br>Category with lax associativity and unit laws   |

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|----|-----------------------------------------------------------------------------|------------------------------------------------------------------------|--------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|
| 17 | Continuous Lattice: Lattice with certain completeness properties            | Algebraic Lattice: Lattice with certain algebraic properties           | Stone Duality: Duality between topological spaces and lattices           | Scott Domain: Continuous domain theory                                                                              | Fuzzy Set: Generalizes set theory with degrees of membership                                        | Complete Lattice: All sup and inf                                                               |
| 18 | Probabilistic Logic: Logic with probabilistic truth values                  | Temporal Logic: Logic for reasoning about time                         | Dynamic Logic: Logic for reasoning about programs                        | Epistemic Logic: Logic for reasoning about knowledge                                                                | Substructural Logic: Logic without certain structural rules                                         | Higher Order Logic: Quantifies over functions                                                   |
| 19 | TQFT: Topological Quantum Field Theory                                      | Vertex Algebra: Algebraic structure used in conformal field theory     | Tensor Network: Network of tensors used in quantum computing             | AdS/CFT Correspondence: Duality between a gravitational theory in Anti-de Sitter space and a conformal field theory | Quantum Gravity: Study of gravity in the context of quantum mechanics                               | String Theory: Theoretical framework in which particles are replaced by one-dimensional strings |
| 20 | TQFT: Topological Quantum Field Theory                                      | D-brane: Objects in string theory that boundaries of strings attach to | Quantum Cohomology: Cohomology theory using quantum mechanics principles | Seiberg-Witten Theory: Study of 4-manifolds using gauge theory                                                      | Floer Theory: Intersection theory for symplectic manifolds                                          | Gromov-Witten Invariants: Counts of holomorphic curves in symplectic manifolds                  |
| 21 | Lie 2-Algebra: Generalization of Lie algebras with 2-categories             | Lie Groupoid: Generalization of Lie groups to groupoids                | Lie Supergroup: Supergroup structure extending Lie groups                | Loop Group: Infinite-dimensional Lie group associated with loops in a Lie group                                     | Virasoro Algebra: Infinite-dimensional Lie algebra related to conformal field theory                | Kac-Moody Algebra: Generalization of finite-dimensional Lie algebras                            |
| 22 | -Groupoid: Generalization of groupoids to infinite levels                   | Simplicial Set: Combinatorial model for topological spaces             | Cubical Set: Generalization of simplicial sets using cubes               | Kan Complex: Special type of simplicial set used in homotopy theory                                                 | Model Category: Category with a notion of weak equivalences, fibrations, and cofibrations           | Spectra: Sequence of spaces or spectra used in stable homotopy theory                           |
| 23 | Derived Scheme: Generalization of schemes in derived algebraic geometry     | Derived Stack: Generalization of stacks in derived algebraic geometry  | Perfectoid Space: Topological space used in p-adic Hodge theory          | Berkovich Space: Non-Archimedean analytic space                                                                     | Tropical Geometry: Study of piecewise linear structures that arise as limits of algebraic varieties | Non-Archimedean Geometry: Study of spaces with non-Archimedean metrics                          |
| 24 | Higher Topos Theory: Study of higher categories with topological structures | (,1)-Category: Category with weak equivalences and higher morphisms    | (,n)-Category: Generalization of (,1)-categories to n-levels             | -Topos: Generalization of topoi to -categories                                                                      | Double Category: Category with two types of morphisms                                               | Tricategory: Generalization of bicategories to three levels                                     |

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| 25 | Derived Category of Sheaves: Category of complexes of sheaves            | Tate Cohomology: Cohomology theory for finite groups                                   | Adams Spectral Sequence: Tool for calculating homotopy groups                                      | Etale Cohomology: Cohomology theory for algebraic varieties                                     | Crystalline Cohomology: Cohomology theory for p-adic varieties                     | De Rham Cohomology: Cohomology for smooth manifolds                                      |
| 26 | Geometric Representation Theory: Study of representations using geometry | Categorical Representation Theory: Study of representations in higher categories       | Quantum Group Representation: Study of representations of quantum groups                           | Hecke Algebra Representation: Study of representations of Hecke algebras                        | Affine Lie Algebra Representation: Study of representations of affine Lie algebras | Kac-Moody Algebra Representation: Study of representations of Kac-Moody algebras         |
| 27 | Symmetric Tensor Category: Category with a symmetric tensor product      | Braided Tensor Category: Category with a braided tensor product                        | Fusion Category: Tensor category with additional properties                                        | Modular Tensor Category: Fusion category satisfying modularity conditions                       | Factorization Algebra: Algebraic structure used in quantum field theory            | Vertex Algebra: Algebraic structure used in conformal field theory                       |
| 28 | Homotopy Type Theory: Homotopy theory interpreted in type theory         | Stable Homotopy Theory: Study of stable homotopy groups of spheres                     | Chromatic Homotopy Theory: Study of periodic phenomena in stable homotopy theory                   | Motivic Homotopy Theory: Homotopy theory for algebraic varieties                                | Equivariant Homotopy Theory: Homotopy theory considering group actions             | Algebraic K-Theory: Study of projective modules and vector bundles using homotopy theory |
| 29 | Quantum Field Theory: Study of quantum fields                            | Conformal Field Theory: Quantum field theory invariant under conformal transformations | Topological Quantum Field Theory: Quantum field theory invariant under topological transformations | String Theory: Theoretical framework in which particles are replaced by one-dimensional strings | M-Theory: Theory unifying the five superstring theories                            | Loop Quantum Gravity: Quantum theory of gravity                                          |
| 30 | Calabi-Yau Manifold: Ricci-flat Kähler manifold                          | Kähler Geometry: Study of Kähler manifolds                                             | Hermitian Symmetric Space: Symmetric space with Hermitian structure                                | Teichmüller Space: Space of complex structures on a surface                                     | Moduli Space of Abelian Varieties: Parameter space for Abelian varieties           | Differential Topology: Study of differentiable functions on manifolds                    |
| 31 | Loop Space: Space of loops in a topological space                        | Operad: Abstract structure for operations                                              | PROPs: Generalization of operads                                                                   | Monoidal Homotopy Theory: Study of homotopy theory with monoidal structures                     | Higher Bordism Theory: Study of manifolds up to cobordism                          | Higher Homotopy Groups: Study of higher homotopy groups                                  |

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| 32 | Symplectic Manifold: Manifold with a non-degenerate 2-form                                   | Contact Manifold: Odd-dimensional counterpart to symplectic manifolds                | Poisson Manifold: Generalizes symplectic manifolds with Poisson brackets                 | Hamiltonian Dynamics: Study of dynamical systems governed by Hamiltonian functions  | Quantum Symplectic Geometry: Study of symplectic geometry in the quantum context                             | Symplectic Field Theory: Generalization of Floer homology                   |
| 33 | Noncommutative Algebraic Geometry: Generalizes algebraic geometry using noncommutative rings | Noncommutative Topology: Study of topological structures in a noncommutative context | Noncommutative Probability: Generalizes probability theory using noncommutative algebras | Quantum Groups: Algebraic structures underlying quantum symmetries                  | C*-Algebra: Study of Banach algebras with an involution                                                      | von Neumann Algebra: Algebra of bounded operators                           |
| 34 | Topological Field Theory: Field theory invariant under topological transformations           | Knot Theory: Study of mathematical knots                                             | Quantum Invariants: Invariants of knots and 3-manifolds from quantum field theory        | Floer Homology: Homology theory for studying periodic orbits in symplectic geometry | Quantum Topology: Study of topological properties in the quantum context                                     | Quantum Field Theory: Study of quantum fields                               |
| 35 | Differential Forms: Generalized functions for integration on manifolds                       | De Rham Cohomology: Cohomology theory using differential forms                       | Riemannian Geometry: Study of smooth manifolds with Riemannian metrics                   | Lorentzian Geometry: Study of manifolds with Lorentzian metrics                     | Finsler Geometry: Generalizes Riemannian geometry by relaxing the requirement for the metric to be quadratic | Complex Geometry: Study of complex manifolds and their properties           |
| 36 | Algebraic K-Theory: Study of projective modules and vector bundles using homotopy theory     | Quillen K-Theory: Quillen's approach to algebraic K-theory                           | Higher Algebraic K-Theory: Generalizes algebraic K-theory to higher dimensions           | Motivic K-Theory: Study of algebraic K-theory in the motivic context                | Topological K-Theory: Study of vector bundles on topological spaces                                          | Higher Topological K-Theory: Generalization of topological K-theory         |
| 37 | Banach Space: Complete normed vector space                                                   | Hilbert Space: Complete inner product space                                          | Operator Algebras: Study of algebras of operators on a Hilbert space                     | Spectral Theory: Study of the spectrum of operators                                 | Functional Calculus: Study of functions of operators                                                         | Banach Algebra: Normed algebra                                              |
| 38 | Geometric Langlands Program: Geometric approach to the Langlands program                     | Derived Categories of Sheaves: Study of derived categories in the context of sheaves | Perverse Sheaves: Special type of sheaves used in representation theory                  | D-Modules: Modules over the ring of differential operators                          | Automorphic Forms: Generalization of modular forms to more general groups                                    | Langlands Program: Conjectures relating Galois groups and automorphic forms |

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| 39 | Arithmetic Geometry: Study of solutions to polynomial equations with integer coefficients | Iwasawa Theory: Study of number fields and Galois representations                            | Modular Forms: Complex analytic functions with specific transformation properties      | Elliptic Curves: Study of cubic curves with a group structure                                       | L-Functions: Complex functions associated with number fields and modular forms          | Class Field Theory: Study of abelian extensions of number fields                  |
| 40 | Computational Group Theory: Study of algorithms for groups                                | Computational Number Theory: Study of algorithms for number theoretic problems               | Computational Algebraic Geometry: Study of algorithms for algebraic geometry problems  | Symbolic Computation: Study of algorithms for manipulating symbolic expressions                     | Computational Topology: Study of algorithms for topological problems                    | Computational Complexity: Study of the complexity of algorithms                   |
| 41 | Linear Programming: Optimization of linear functions subject to linear constraints        | Integer Programming: Optimization of linear functions with integer variables                 | Combinatorial Optimization: Study of optimization problems on combinatorial structures | Network Flow: Study of flows in networks                                                            | Polyhedral Combinatorics: Study of combinatorial properties of polyhedra                | Graph Theory: Study of graphs and their properties                                |
| 42 | Algebraic Graph Theory: Study of graphs using algebraic methods                           | Combinatorial Design Theory: Study of combinatorial designs                                  | Matroid Theory: Study of matroids and their applications                               | Symmetric Functions: Study of symmetric functions and their applications                            | Representation Theory of Symmetric Groups: Study of representations of symmetric groups | Representation Theory of Finite Groups: Study of representations of finite groups |
| 43 | Homological Algebra: Study of homology and cohomology theories                            | Derived Functors: Functors constructed to measure the failure of another functor to be exact | Ext and Tor: Derived functors measuring extensions and torsion                         | Derived Categories: Categories of complexes of objects                                              | Triangulated Categories: Categories equipped with a class of distinguished triangles    | Derived Algebraic Geometry: Study of derived categories in algebraic geometry     |
| 44 | Morse Theory: Study of critical points of smooth functions                                | Surgery Theory: Study of the modifications of manifolds                                      | Cobordism Theory: Study of manifolds up to cobordism                                   | Differential Structures on Manifolds: Study of different smooth structures on topological manifolds | Exotic Spheres: Study of spheres with exotic smooth structures                          | Homotopy Theory: Study of homotopies and their properties                         |
| 45 | Ergodic Theory: Study of dynamical systems with an invariant measure                      | Measure Theory: Study of measures and their properties                                       | Probability Theory: Study of random events and their probabilities                     | Dynamical Systems: Study of systems that evolve over time                                           | Statistical Mechanics: Study of large systems by statistical methods                    | Stochastic Processes: Study of processes that evolve randomly over time           |

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| 46 | Axiomatic Set Theory: Study of sets based on axioms                           | Large Cardinals: Study of large cardinal numbers                                          | Descriptive Set Theory: Study of sets in descriptive terms               | Forcing: Technique to prove consistency and independence results                                    | Inner Models: Study of models of set theory inside other models                       | Infinitary Combinatorics: Study of combinatorial properties of infinite sets                |
| 47 | Model Theory: Study of models of formal theories                              | O-Minimal Structures: Study of structures with a well-behaved definable set               | Stability Theory: Study of stable theories and their models              | NIP Theories: Study of theories without the independence property                                   | Categoricity: Study of categorical theories                                           | Definability: Study of definable sets and functions                                         |
| 48 | Proof Theory: Study of the structure of mathematical proofs                   | Sequent Calculus: A logic system for proving validity                                     | Natural Deduction: A method for deriving logical conclusions             | Type Theory: Study of types in programming languages and logic                                      | Automated Theorem Proving: Study of algorithms for proving theorems                   | Lambda Calculus: Computation with functions                                                 |
| 49 | Derived Algebraic Geometry: Study of derived categories in algebraic geometry | Noncommutative Algebraic Geometry: Study of algebraic geometry using noncommutative rings | Logarithmic Geometry: Study of log schemes and their applications        | Tropical Geometry: Study of piecewise linear structures that arise as limits of algebraic varieties | Homotopical Algebraic Geometry: Study of algebraic geometry using homotopical methods | Higher Category Theory: Study of categories with higher-dimensional morphisms               |
| 50 | Geometric Representation Theory: Study of representations using geometry      | Categorical Representation Theory: Study of representations in higher categories          | Quantum Group Representation: Study of representations of quantum groups | Hecke Algebra Representation: Study of representations of Hecke algebras                            | Affine Lie Algebra Representation: Study of representations of affine Lie algebras    | Kac-Moody Algebra Representation: Study of representations of Kac-Moody algebras            |
| 51 | Quantum Mechanics: Study of the behavior of particles at the quantum level    | Quantum Field Theory: Study of quantum fields                                             | Quantum Gravity: Study of gravity in the context of quantum mechanics    | Quantum Computing: Study of computation using quantum-mechanical phenomena                          | Quantum Cryptography: Study of cryptographic systems using quantum mechanics          | Quantum Information Theory: Study of information theory in the context of quantum mechanics |
| 52 | Graph Theory: Study of graphs and their properties                            | Hypergraph Theory: Study of hypergraphs and their properties                              | Matroid Theory: Study of matroids and their applications                 | Combinatorial Design Theory: Study of combinatorial designs                                         | Finite Geometry: Study of finite geometric structures                                 | Algebraic Combinatorics: Study of combinatorial structures using algebraic methods          |

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| 53 | Algebraic Number Theory: Study of algebraic structures related to algebraic numbers | Class Field Theory: Study of abelian extensions of number fields     | Iwasawa Theory: Study of number fields and Galois representations                | Modular Forms: Complex analytic functions with specific transformation properties | L-Functions: Complex functions associated with number fields and modular forms                  | Diophantine Geometry: Study of Diophantine equations                      |
| 54 | Nonlinear Dynamics: Study of systems governed by nonlinear equations                | Chaos Theory: Study of chaotic systems and their properties          | Bifurcation Theory: Study of changes in the structure of dynamical systems       | Fractal Geometry: Study of fractals and their properties                          | Soliton Theory: Study of solitons and their interactions                                        | Dynamical Systems: Study of systems that evolve over time                 |
| 55 | Topological Data Analysis: Study of the shape of data using topology                | Persistent Homology: Study of homology over varying scales           | Mapper Algorithm: Technique for topological data analysis                        | Sheaf Theory: Study of sheaves and their applications                             | Discrete Morse Theory: Study of discrete analogs of Morse theory                                | Computational Topology: Study of algorithms for topological problems      |
| 56 | Mathematical Physics: Study of mathematical methods in physics                      | Statistical Mechanics: Study of large systems by statistical methods | Quantum Mechanics: Study of the behavior of particles at the quantum level       | General Relativity: Study of gravitation in the context of relativity             | String Theory: Theoretical framework in which particles are replaced by one-dimensional strings | Quantum Field Theory: Study of quantum fields                             |
| 57 | Computational Geometry: Study of algorithms for geometric problems                  | Geometric Modeling: Study of the representation of geometric shapes  | Mesh Generation: Study of algorithms for generating meshes                       | Voronoi Diagrams: Study of Voronoi diagrams and their applications                | Convex Hulls: Study of convex hulls and their properties                                        | Triangulations: Study of the division of geometric objects into simplices |
| 58 | Geometric Group Theory: Study of groups using geometric methods                     | Hyperbolic Geometry: Study of hyperbolic spaces and their properties | CAT(0) Spaces: Study of spaces with non-positive curvature                       | Teichmüller Theory: Study of the Teichmüller space of a surface                   | Geometric Invariant Theory: Study of group actions on algebraic varieties                       | Low-Dimensional Topology: Study of topological spaces of low dimension    |
| 59 | Homotopy Theory: Study of homotopies and their properties                           | Stable Homotopy Theory: Study of stable homotopy groups of spheres   | Chromatic Homotopy Theory: Study of periodic phenomena in stable homotopy theory | Motivic Homotopy Theory: Homotopy theory for algebraic varieties                  | Equivariant Homotopy Theory: Homotopy theory considering group actions                          | Homotopical Algebra: Study of algebraic structures using homotopy theory  |
| 60 | Algebraic Topology: Study of topological spaces with algebraic methods              | Homology Theory: Study of homology and cohomology theories           | Cohomology Theory: Study of cohomology theories                                  | Spectral Sequences: Tools for computing homology and cohomology groups            | Higher Homotopy Groups: Study of higher homotopy groups                                         | Fiber Bundles: Study of bundles with fibers over a base space             |

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| 61 | Probability Theory: Study of random events and their probabilities                          | Stochastic Processes: Study of processes that evolve randomly over time                      | Martingales: Study of martingales and their properties                               | Markov Chains: Study of Markov chains and their properties                                                   | Brownian Motion: Study of Brownian motion and its properties         | Measure Theory: Study of measures and their properties                       |
| 62 | Riemannian Geometry: Study of smooth manifolds with Riemannian metrics                      | Lorentzian Geometry: Study of manifolds with Lorentzian metrics                              | Symplectic Geometry: Study of manifolds with a symplectic structure                  | Finsler Geometry: Generalizes Riemannian geometry by relaxing the requirement for the metric to be quadratic | Complex Geometry: Study of complex manifolds and their properties    | Algebraic Geometry: Study of solutions to polynomial equations               |
| 63 | Topological Groups: Study of groups with a topology                                         | Topological Vector Spaces: Study of vector spaces with a topology                            | Fiber Bundles: Study of bundles with fibers over a base space                        | Principal Bundles: Study of bundles with a group action                                                      | Sheaf Theory: Study of sheaves and their applications                | Homotopy Theory: Study of homotopies and their properties                    |
| 64 | Noncommutative Geometry: Study of geometric structures arising from noncommutative algebras | Noncommutative Algebraic Geometry: Generalizes algebraic geometry using noncommutative rings | Noncommutative Topology: Study of topological structures in a noncommutative context | Noncommutative Probability: Generalizes probability theory using noncommutative algebras                     | Quantum Groups: Algebraic structures underlying quantum symmetries   | Hopf Algebras: Bialgebra with an antipode                                    |
| 65 | Ergodic Theory: Study of dynamical systems with an invariant measure                        | Measure Theory: Study of measures and their properties                                       | Probability Theory: Study of random events and their probabilities                   | Dynamical Systems: Study of systems that evolve over time                                                    | Statistical Mechanics: Study of large systems by statistical methods | Stochastic Processes: Study of processes that evolve randomly over time      |
| 66 | Axiomatic Set Theory: Study of sets based on axioms                                         | Large Cardinals: Study of large cardinal numbers                                             | Descriptive Set Theory: Study of sets in descriptive terms                           | Forcing: Technique to prove consistency and independence results                                             | Inner Models: Study of models of set theory inside other models      | Infinitary Combinatorics: Study of combinatorial properties of infinite sets |
| 67 | Model Theory: Study of models of formal theories                                            | O-Minimal Structures: Study of structures with a well-behaved definable set                  | Stability Theory: Study of stable theories and their models                          | NIP Theories: Study of theories without the independence property                                            | Categoricity: Study of categorical theories                          | Definability: Study of definable sets and functions                          |

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|----|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------|----------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|
| 68 | Proof Theory:<br>Study of the structure of mathematical proofs                      | Sequent Calculus: A logic system for proving validity                                     | Natural Deduction: A method for deriving logical conclusions               | Type Theory: Study of types in programming languages and logic                                      | Automated Theorem Proving: Study of algorithms for proving theorems                   | Lambda Calculus: Computation with functions                                                 |
| 69 | Derived Algebraic Geometry: Study of derived categories in algebraic geometry       | Noncommutative Algebraic Geometry: Study of algebraic geometry using noncommutative rings | Logarithmic Geometry: Study of log schemes and their applications          | Tropical Geometry: Study of piecewise linear structures that arise as limits of algebraic varieties | Homotopical Algebraic Geometry: Study of algebraic geometry using homotopical methods | Higher Category Theory: Study of categories with higher-dimensional morphisms               |
| 70 | Geometric Representation Theory: Study of representations using geometry            | Categorical Representation Theory: Study of representations in higher categories          | Quantum Group Representation: Study of representations of quantum groups   | Hecke Algebra Representation: Study of representations of Hecke algebras                            | Affine Lie Algebra Representation: Study of representations of affine Lie algebras    | Kac-Moody Algebra Representation: Study of representations of Kac-Moody algebras            |
| 71 | Quantum Mechanics: Study of the behavior of particles at the quantum level          | Quantum Field Theory: Study of quantum fields                                             | Quantum Gravity: Study of gravity in the context of quantum mechanics      | Quantum Computing: Study of computation using quantum-mechanical phenomena                          | Quantum Cryptography: Study of cryptographic systems using quantum mechanics          | Quantum Information Theory: Study of information theory in the context of quantum mechanics |
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| 74 | Nonlinear Dynamics: Study of systems governed by nonlinear equations                | Chaos Theory: Study of chaotic systems and their properties                               | Bifurcation Theory: Study of changes in the structure of dynamical systems | Fractal Geometry: Study of fractals and their properties                                            | Soliton Theory: Study of solitons and their interactions                              | Dynamical Systems: Study of systems that evolve over time                                   |

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|----|------------------------------------------------------------------------|-------------------------------------------------------------------------|----------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------|
| 75 | Topological Data Analysis: Study of the shape of data using topology   | Persistent Homology: Study of homology over varying scales              | Mapper Algorithm: Technique for topological data analysis                        | Sheaf Theory: Study of sheaves and their applications                                                        | Discrete Morse Theory: Study of discrete analogs of Morse theory                                | Computational Topology: Study of algorithms for topological problems      |
| 76 | Mathematical Physics: Study of mathematical methods in physics         | Statistical Mechanics: Study of large systems by statistical methods    | Quantum Mechanics: Study of the behavior of particles at the quantum level       | General Relativity: Study of gravitation in the context of relativity                                        | String Theory: Theoretical framework in which particles are replaced by one-dimensional strings | Quantum Field Theory: Study of quantum fields                             |
| 77 | Computational Geometry: Study of algorithms for geometric problems     | Geometric Modeling: Study of the representation of geometric shapes     | Mesh Generation: Study of algorithms for generating meshes                       | Voronoi Diagrams: Study of Voronoi diagrams and their applications                                           | Convex Hulls: Study of convex hulls and their properties                                        | Triangulations: Study of the division of geometric objects into simplices |
| 78 | Geometric Group Theory: Study of groups using geometric methods        | Hyperbolic Geometry: Study of hyperbolic spaces and their properties    | CAT(0) Spaces: Study of spaces with non-positive curvature                       | Teichmüller Theory: Study of the Teichmüller space of a surface                                              | Geometric Invariant Theory: Study of group actions on algebraic varieties                       | Low-Dimensional Topology: Study of topological spaces of low dimension    |
| 79 | Homotopy Theory: Study of homotopies and their properties              | Stable Homotopy Theory: Study of stable homotopy groups of spheres      | Chromatic Homotopy Theory: Study of periodic phenomena in stable homotopy theory | Motivic Homotopy Theory: Homotopy theory for algebraic varieties                                             | Equivariant Homotopy Theory: Homotopy theory considering group actions                          | Homotopical Algebra: Study of algebraic structures using homotopy theory  |
| 80 | Algebraic Topology: Study of topological spaces with algebraic methods | Homology Theory: Study of homology and cohomology theories              | Cohomology Theory: Study of cohomology theories                                  | Spectral Sequences: Tools for computing homology and cohomology groups                                       | Higher Homotopy Groups: Study of higher homotopy groups                                         | Fiber Bundles: Study of bundles with fibers over a base space             |
| 81 | Probability Theory: Study of random events and their probabilities     | Stochastic Processes: Study of processes that evolve randomly over time | Martingales: Study of martingales and their properties                           | Markov Chains: Study of Markov chains and their properties                                                   | Brownian Motion: Study of Brownian motion and its properties                                    | Measure Theory: Study of measures and their properties                    |
| 82 | Riemannian Geometry: Study of smooth manifolds with Riemannian metrics | Lorentzian Geometry: Study of manifolds with Lorentzian metrics         | Symplectic Geometry: Study of manifolds with a symplectic structure              | Finsler Geometry: Generalizes Riemannian geometry by relaxing the requirement for the metric to be quadratic | Complex Geometry: Study of complex manifolds and their properties                               | Algebraic Geometry: Study of solutions to polynomial equations            |

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| 83 | Topological Groups: Study of groups with a topology                                         | Topological Vector Spaces: Study of vector spaces with a topology                            | Fiber Bundles: Study of bundles with fibers over a base space                        | Principal Bundles: Study of bundles with a group action                                  | Sheaf Theory: Study of sheaves and their applications                                | Homotopy Theory: Study of homotopies and their properties                     |
| 84 | Noncommutative Geometry: Study of geometric structures arising from noncommutative algebras | Noncommutative Algebraic Geometry: Generalizes algebraic geometry using noncommutative rings | Noncommutative Topology: Study of topological structures in a noncommutative context | Noncommutative Probability: Generalizes probability theory using noncommutative algebras | Quantum Groups: Algebraic structures underlying quantum symmetries                   | Hopf Algebras: Bialgebra with an antipode                                     |
| 85 | Ergodic Theory: Study of dynamical systems with an invariant measure                        | Measure Theory: Study of measures and their properties                                       | Probability Theory: Study of random events and their probabilities                   | Dynamical Systems: Study of systems that evolve over time                                | Statistical Mechanics: Study of large systems by statistical methods                 | Stochastic Processes: Study of processes that evolve randomly over time       |
| 86 | Axiomatic Set Theory: Study of sets based on axioms                                         | Large Cardinals: Study of large cardinal numbers                                             | Descriptive Set Theory: Study of sets in descriptive terms                           | Forcing: Technique to prove consistency and independence results                         | Inner Models: Study of models of set theory inside other models                      | Infinitary Combinatorics: Study of combinatorial properties of infinite sets  |
| 87 | Model Theory: Study of models of formal theories                                            | O-Minimal Structures: Study of structures with a well-behaved definable set                  | Stability Theory: Study of stable theories and their models                          | NIP Theories: Study of theories without the independence property                        | Categoricity: Study of categorical theories                                          | Definability: Study of definable sets and functions                           |
| 88 | Proof Theory: Study of the structure of mathematical proofs                                 | Sequent Calculus: A logic system for proving validity                                        | Natural Deduction: A method for deriving logical conclusions                         | Type Theory: Study of types in programming languages and logic                           | Automated Theorem Proving: Study of algorithms for proving theorems                  | Lambda Calculus: Computation with functions                                   |
| 89 | Homological Algebra: Study of homology and cohomology theories                              | Derived Functors: Functors constructed to measure the failure of another functor to be exact | Ext and Tor: Derived functors measuring extensions and torsion                       | Derived Categories: Categories of complexes of objects                                   | Triangulated Categories: Categories equipped with a class of distinguished triangles | Derived Algebraic Geometry: Study of derived categories in algebraic geometry |
| 90 | Mathematical Logic: Study of formal logical systems                                         | Set Theory: Study of sets and their properties                                               | Model Theory: Study of models of formal theories                                     | Proof Theory: Study of the structure of mathematical proofs                              | Recursion Theory: Study of computable functions and Turing degrees                   | Constructivist Logic: Logic based on constructive mathematics                 |

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| 91 | Algebraic Geometry: Study of solutions to polynomial equations             | Commutative Algebra: Study of commutative rings and their ideals        | Homological Algebra: Study of homology and cohomology theories                                  | Representation Theory: Study of algebraic structures through their representations | Category Theory: Study of mathematical structures through categories                     | Higher Category Theory: Study of categories with higher-dimensional morphisms        |
| 92 | Differential Geometry: Study of geometry using differential calculus       | Algebraic Geometry: Study of solutions to polynomial equations          | Riemannian Geometry: Study of smooth manifolds with Riemannian metrics                          | Symplectic Geometry: Study of manifolds with a symplectic structure                | Topology: Study of topological spaces and continuous functions                           | Geometric Topology: Study of manifolds and topological spaces with geometric methods |
| 93 | Combinatorics: Study of counting, arrangement, and combination             | Graph Theory: Study of graphs and their properties                      | Matroid Theory: Study of matroids and their applications                                        | Design Theory: Study of combinatorial designs                                      | Finite Geometry: Study of finite geometric structures                                    | Enumerative Combinatorics: Study of counting combinatorial structures                |
| 94 | Numerical Analysis: Study of algorithms for numerical approximation        | Computational Geometry: Study of algorithms for geometric problems      | Symbolic Computation: Study of algorithms for manipulating symbolic expressions                 | Cryptography: Study of secure communication techniques                             | Complexity Theory: Study of the complexity of algorithms                                 | Computational Number Theory: Study of algorithms for number theoretic problems       |
| 95 | Real Analysis: Study of real-valued functions and sequences                | Complex Analysis: Study of complex-valued functions and sequences       | Functional Analysis: Study of vector spaces with infinite dimensions                            | Harmonic Analysis: Study of functions and signals using Fourier series             | Nonlinear Analysis: Study of nonlinear equations and their solutions                     | Geometric Analysis: Study of geometry using differential and integral calculus       |
| 96 | Probability Theory: Study of random events and their probabilities         | Stochastic Processes: Study of processes that evolve randomly over time | Martingales: Study of martingales and their properties                                          | Markov Chains: Study of Markov chains and their properties                         | Brownian Motion: Study of Brownian motion and its properties                             | Measure Theory: Study of measures and their properties                               |
| 97 | Quantum Mechanics: Study of the behavior of particles at the quantum level | General Relativity: Study of gravitation in the context of relativity   | String Theory: Theoretical framework in which particles are replaced by one-dimensional strings | Quantum Field Theory: Study of quantum fields                                      | Statistical Mechanics: Study of large systems by statistical methods                     | Quantum Gravity: Study of gravity in the context of quantum mechanics                |
| 98 | Applied Mathematics: Application of mathematical methods to other fields   | Mathematical Biology: Application of mathematics to biological problems | Mathematical Physics: Application of mathematics to physical problems                           | Financial Mathematics: Application of mathematics to financial problems            | Computational Mathematics: Application of computational methods to mathematical problems | Industrial Mathematics: Application of mathematics to industrial problems            |

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| 99  | Mathematical Neuroscience:<br>Application of mathematics to neuroscience                                      | Mathematical Epidemiology:<br>Application of mathematics to the study of epidemics | Mathematical Ecology:<br>Application of mathematics to ecological problems           | Mathematical Economics:<br>Application of mathematics to economic problems    | Mathematical Psychology:<br>Application of mathematics to psychological problems | Mathematical Sociology:<br>Application of mathematics to sociological problems |
| 100 | Topological Quantum Field Theory: Study of quantum field theories invariant under topological transformations | Higher Category Theory: Study of categories with higher-dimensional morphisms      | Higher Algebraic Geometry: Generalization of algebraic geometry to higher dimensions | Higher Symplectic Geometry: Study of symplectic geometry in higher dimensions | Higher Representation Theory: Study of representations in higher categories      | Higher Homotopy Theory: Study of homotopy theory in higher dimensions          |